Inverse Algorithms for the GPR Assessment of Concrete Structures

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***Abstract*— In this work, we use two different approaches for processing GPR data by means of linear inverse scattering. In order to identify the characteristics of the host medium and its inclusions a Particle Swarm Optimization algorithm is used. Focusing is done using a Matched-Filter-Based Reverse-Time migration algorithm in order to decrease the dependence upon the personal expertise of the human operator.**

***Index Terms*— FDTD, Inverse Scattering, Migration algorithms, Particle Swarm Optimization.**

I. Introduction

NONDESTRUCTIVE inspection of concrete structures using radar techniques is increasingly being recognised as an effective way of gathering information. The numerical simulation of this type of inspection may help to minimize the overall cost of an investigation and to increase the likelihood of carrying out fully effective maintenance and repair. In this work, the finite-difference time-domain (FDTD) technique is applied to simulate the radar assessment of concrete structures with PVC ducts. The research aim of the numerical experiments is the location of voids inside concrete structures using linear inverse algorithms.

II. The inverse scattering problem

An incident wave and a scattered wave can be used to characterize the scattering object. Usually in real world problems the incident and scattered waves are know and it is desired to identify the scattering object. This is called the inverse scattering problem. It can be written as an optimization problem involving the scattered wave of the unknown object $E(\theta_0)$, the reference object, and the scattered wave of a test object $E(\theta)$. Thus, θ^*, the optimum θ, is the argument that minimizes the error of the reference object scattered wave $E(\theta_0)$ relative to the test object scattered wave $E(\theta)$. Mathematically:

$$\theta^* = \ arg\ min\ f(\theta) = \sum_{i=1}^{ns} (E(\theta_0) - E(\theta))^2 \qquad (1)$$

where ns are the sample points where the scattered wave is measured. Note that $E(\theta_0)$ is known even though θ_0 is unknown, it is the measure on the receiver antennas. The scattered $E(\theta)$ is then generated assuming one test θ, and the optimization procedure aims at minimizing the error between $E(\theta_0)$ and $E(\theta)$ in such a way as to identify the scatter object θ_0. This paper studies a $2D$ inverse scattering problem with non homogenous media trying to identify the standard deviation of the permittivity, sd, the depth of the inclusion and the inclusion radii. This problem was solved using the Particle Swarm Optimization, and is described next.

III. Particle Swarm Optimization

The Particle Swarm Optimization (PSO) is a stochastic evolutionary technique dating to 1995 [1], [2]. The PSO is similar to genetic algorithms (GAs) due to the random initialization. The first difference is that each potential solution is called particles, instead of individuals, and they ”fly” on the search space. To each particle of the swarm during the iterations, the positions of the best solution found to a given particle, called $pbest$ (particle best) is saved. The best value found considering all the particle is also saved, and is called $gbest$ (global best). At each iteration, the PSO is based on the change in the particle's velocity in the direction of its $pbest$ and $gbest$, weighted by a random term. The PSO, as originally described, is as follows:

1) Initialize the swarm of particles with random positions and velocities.
2) For each particle calculate the objective function.
3) Find $pbest$.
4) Find $gbest$.
5) Change the velocity and position of each particle according to Eqs. 2 and 3

$$v = v + c_1 * rand * (\textbf{pbest} - x) + c_2 * rand * (\textbf{gbest} - x) \qquad (2)$$

$$x = x + v \qquad (3)$$

6) Return to step 2 untill one stop criteria is achieved.

The velocity of each particle in each dimension is limited by a maximum velocity, $Vmax$.

The acceleration constants c_1 and c_2 used in Eq. 2, represent the trade-off between the search in the direction of $pbest$ and $gbest$. Usual values to c_1 and c_2 are equal to 2 and $Vmax$ between 10% and 20% of the variable range in each dimension.

IV. Migration Algorithm

To improve the interpretation of the GPR assessment a reverse-time migration technique [3] to find the exact location of the targets in heterogeneous media was implemented. The development of this algorithm is based on the notion of a matched filter, which is used extensively in radar applications. Using this algorithm, an image can be perceived as a back propagated wave-field reconstruction of the dielectric contrast within the host medium [3]. The final migrated data for a bistatic configuration can be obtained by the following equation:

$$S(\vec{r}) = \sum_{n=1}^{N} \sum_{m=1}^{M} E_{mn,bp}(\vec{r}) \otimes E_{inc}(\vec{r})\,|_{t=0} \,. \qquad (4)$$

where the subscripts *m* and *n* denote the field due to the *m*th transmitter and *n*th receiver. The bistatic algorithm requires propagation of both the incident and back-propagated fields. The data were collected at 60 locations. The implementation in FDTD is accomplished by propagating the incident field in reverse while simultaneously propagating the back propagated field forward.

V. Results

For the two approaches the problem geometry consisting of two half spaces depicted in Figure 1 is used. The antennas are placed in free space above an nonhomogeneous dielectric.

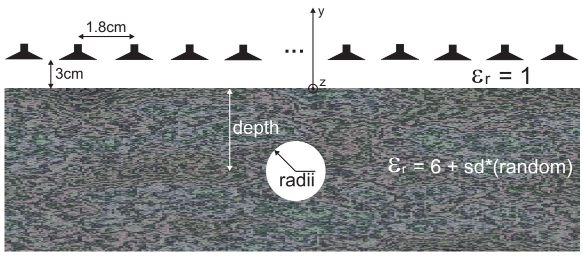

Fig. 1. Configuration consisting of a circular cylinder located in nonhomogeneous dieletric.

A. Particle Swarm Optimization

The unknown object was defined in our experiment with the depth of the inclusion equal to $10cm$, the radii equal to $4.5cm$ and the standard deviation of the nonhomogenous media, sd, equal to 0.15. The considered target was water. The definition of the reference object as well as the range of the variables for the optimization process are summarized in Table I.

TABLE I
Inverse scattering problem definitions

	Depth	Radii	Sd
Min	$5cm$	$2.5cm$	0.05
Max	$25cm$	$10cm$	0.30
Ref. Object	$10cm$	$4.5cm$	0.15

The PSO was initialized using 50 particles over 50 iterations. The $gbest$ (global best) evolution through out the iterations is shown in Table II. In the $50th$ iteration, the algorithm it was capable of finding a solution very close to the desired one. Nonetheless, good approximations were already know in the $10th$ iteration, thus, if a very precise outcome is not necessary the algorithm would need about only about 10 iterations to converge.

TABLE II
PSO results

Iteration	Depth	Radii	Sd
1	$8.7cm$	$3.2cm$	0.297
5	$9.7cm$	$4.4cm$	0.156
10	$9.7cm$	$4.7cm$	0.149
50	$10cm$	$4.44cm$	0.15
Ref. Object	$10cm$	$4.5cm$	0.15

B. Migration algorithm

The FDTD scenario simulated consisted of four 12-cm diameter PVC pipes buried in concrete that was modeled with a mean relative electrical permittivity value of 6, conductivity $1mS/m$ and standard deviation 0.25. Figure 2 shows the final image obtained at $t = 0$ that can be interpreted as the intersection of the back propagated field with the incident field providing a more exact location of the hollow tubes. In conclusion, this process improves the final images provided by the radar inspection data. However, this algorithm assumes that the background medium is known. In addition, in this simulation, line sources were used which can create a problem when the objective is to find targets in close proximity.

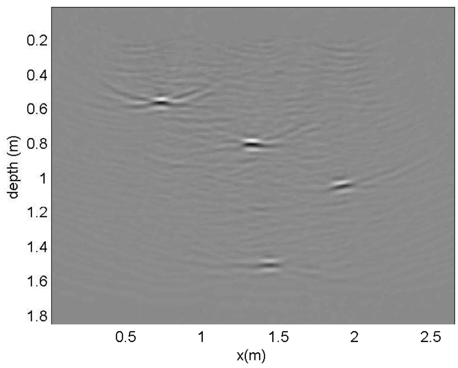

Fig. 2. Bistatic reverse-time migration; final data

VI. Conclusion

The problem of inverting GPR data has been investigated using two different approaches. The PSO provided a precise answer characterizing the target and the degree of heterogeneity of the host medium. However to do that 50 iterations were necessary. The migration algorithm instead can be performed using only one iteration requiring only the permittivity of the host medium. In future work we plan to mix these two approaches in order to get more information about the assessment using as few iterations as possible.

References

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